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MINIMAL LOCAL LAGRANGIANS FOR HIGHER-SPIN GEOMETRY

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ABSTRACT

The Fronsdal Lagrangians for free totally symmetric rank- s tensors $\varphi_{\mu_1 \dots \mu_s}$ rest on suitable trace constraints for their gauge parameters and gauge fields. Only when these constraints are removed, however, the resulting equations reflect the expected free higher-spin geometry. We show that geometric equations, in both their local and non-local forms, can be simply recovered from local Lagrangians with *only two* additional fields, a rank- $(s-3)$ compensator $\alpha_{\mu_1 \dots \mu_{s-3}}$ and a rank- $(s-4)$ Lagrange multiplier $\beta_{\mu_1 \dots \mu_{s-4}}$. In a similar fashion, we show that geometric equations for unconstrained rank- n totally symmetric spinor-tensors $\psi_{\mu_1 \dots \mu_n}$ can be simply recovered from local Lagrangians with *only two* additional spinor-tensors, a rank- $(n-2)$ compensator $\xi_{\mu_1 \dots \mu_{n-2}}$ and a rank- $(n-3)$ Lagrange multiplier $\lambda_{\mu_1 \dots \mu_{n-3}}$.

1 INTRODUCTION AND SUMMARY

Higher-spin gauge fields¹ are an old and fascinating corner of Field Theory, with many open questions, some of which appear of direct relevance for a deeper understanding of String Theory. Only a few Lorentz representations play a role in the present models of the Fundamental Interactions (two-tensors, vectors, scalars, spinors, and often their super-partners), while the term “higher spin” qualifies in principle fields belonging to all types of representations that are more complicated than these. In practice, however, one often restricts the attention to rank- s totally symmetric tensors $\varphi_{\mu_1 \dots \mu_s}$, that generalize the metric tensor fluctuation $h_{\mu\nu}$ and possess the gauge transformations

$$\delta \varphi_{\mu_1 \dots \mu_s} = \partial_{\mu_1} \Lambda_{\mu_2 \dots \mu_s} + \dots, \quad (1.1)$$

with parameters $\Lambda_{\mu_1 \dots \mu_{s-1}}$ that are themselves totally symmetric tensors, of rank- $(s-1)$, or to rank- n totally symmetric spinor-tensors $\psi_{\mu_1 \dots \mu_n}$, that generalize the gravitino field ψ_μ of supergravity and possess the gauge transformations

$$\delta \psi_{\mu_1 \dots \mu_n} = \partial_{\mu_1} \epsilon_{\mu_2 \dots \mu_n} + \dots, \quad (1.2)$$

with parameters $\epsilon_{\mu_1 \dots \mu_{n-1}}$ that are themselves totally symmetric spinor-tensors, of rank- $(n-1)$. For this class of higher-spin fields, explicit statements can often be made efficiently and concisely for arbitrary values of s or n . It should be borne in mind, however, that these types of fields do not exhaust all available possibilities in more than four dimensions, and indeed tensors of mixed symmetry are an important part of the massive spectrum of String Theory. Previous results concerning the free theory have been consistently generalized to cases of mixed symmetry, albeit necessarily in a less explicit fashion, and hence, for the sake of clarity, here we shall restrict ourselves to the totally symmetric case, leaving for future work a detailed analysis of the more involved cases of mixed symmetry. Abiding to common practice, from now on we shall simply use the term “spin” for the rank of bosonic tensors, or for the rank of Fermi fields augmented by $1/2$.

The conventional formulation for free totally symmetric tensor gauge fields $\varphi_{\mu_1 \dots \mu_s}$ was originally deduced by Fronsdal [6], in the late seventies, from the massive Singh-Hagen Lagrangians [7]. The key feature of this formulation is the need for a pair of constraints, one on the gauge parameter, $\Lambda_{\mu_1 \dots \mu_{s-1}}$, whose *trace* $\Lambda'_{\mu_3 \dots \mu_{s-1}} \equiv \eta^{\mu_1 \mu_2} \Lambda_{\mu_1 \dots \mu_{s-1}}$ is required to vanish, and one on the gauge field itself, whose *double trace* $\varphi''_{\mu_5 \dots \mu_s} \equiv \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4} \varphi_{\mu_1 \dots \mu_s}$ is also required to vanish. In a similar fashion, the conventional formulation for free totally symmetric spinor-tensors, due to Fang and Fronsdal [8], requires that both the γ - *trace* $\not\epsilon_{\mu_2 \dots \mu_{n-1}}$ of the gauge parameter and the *triple* γ - *trace*² of the spinor gauge field $\psi'_{\mu_4 \dots \mu_n}$ vanish. While these constraints result in a consistent free dynamics, it is difficult to regard them as natural ingredients of a complete formulation of higher-spin gauge fields.

There is a body of evidence that consistent higher-spin interactions generally require that an infinite number of such fields be mutually coupled. For instance, the gravitational coupling for a single higher-spin field suffers from inconsistencies, the so-called Aragone-Deser [9] problem, that can only be avoided in special cases, and most notably in (A)dS backgrounds. Only in such special circumstances can these fields be considered in isolation as in flat space. On

¹The web site <http://www.ulb.ac.be/sciences/ptm/pmif/Solvay1proc.pdf> contains the Proceedings of the First Solvay Workshop on Higher-Spin Gauge Fields, with a number of contributions, including [1, 2, 3, 4, 5] that are more closely related to this work, and many references to the original literature.

²For symmetric spinor-tensors two γ - traces are equivalent to a trace.

the other hand, interacting systems of higher-spin fields are bound to be very complicated, and hence are not fully under control to date, but possess the intriguing virtue of being of intermediate complexity between ordinary low-spin Field Theory and String Theory. Still, many important results are now available. Most notably, in the presence of a cosmological term, the perturbative definition of higher-spin interactions around (A)dS spaces can avoid the difficulties long recognized for their naive flat-space couplings [10]. This crucial observation led Vasiliev to formulate a consistent set of coupled non-linear equations for totally symmetric tensors, first in four dimensions and, more recently, in arbitrary dimensions as well. The Vasiliev equations [11, 13] (see also [12] for relevant contributions along these lines) represent the most encouraging result on higher-spin gauge fields available to date, although they are clearly non-Lagrangian and more work is required to arrive at an off-shell formulation. They generalize both the frame formulation of Einstein gravity and the Cartan integrable systems that have long emerged from supergravity [14], extending them to allow for non-polynomial scalar couplings.

The Vasiliev equations are based on an extension of the frame formalism for gravity, and as a result their fields, forms valued in representations of the tangent Lorentz group, can simply accommodate Fronsdal's constraints. Still, other possibilities should be explored at this stage, and in metric-like formulations the trace constraints of [6, 8] should naturally be absent. With this motivation in mind, in [15] we showed that it is possible to extend the Fronsdal construction to allow for unconstrained gauge fields and parameters. A nice outcome was the prompt emergence of the geometry underlying the field equations, that become

$$\frac{1}{\square^p} \partial \cdot \mathcal{R}^{[p]};_{\alpha_1 \dots \alpha_{2p+1}} = 0 , \quad (1.3)$$

for odd spins $s = 2p + 1$, and

$$\frac{1}{\square^{p-1}} \mathcal{R}^{[p]};_{\alpha_1 \dots \alpha_{2p}} = 0 , \quad (1.4)$$

for even spins $s = 2p$, where \mathcal{R} denotes the linearized higher-spin curvature introduced by de Wit and Freedman in [16] and the bracketed suffix denotes that p - fold traces are taken. Being *non local* for spin $s > 2$, both these equations and the corresponding Lagrangians are not easy to use, but their geometrical nature is nonetheless quite suggestive. Similar constructions for higher-spin fermions were also presented in [15], and the bosonic construction of [15] was generalized to mixed symmetry tensors in [17].

Local field equations for unconstrained fields can actually be obtained without much effort. Confining our attention for simplicity to bosonic fields, it is possible to show [18, 19] that the non-local geometric equations (1.3) and (1.4) are equivalent to simple non-Lagrangian systems involving a new field, a spin - $(s - 3)$ compensator $\alpha_{\mu_1 \dots \mu_{s-3}}$, that under gauge transformations transforms as

$$\delta \alpha_{\mu_1 \dots \mu_{s-3}} = \Lambda'_{\mu_1 \dots \mu_{s-3}} . \quad (1.5)$$

As we shall review in the next Section, the resulting local compensator form of the higher-spin equations is equivalent to eqs. (1.3) and (1.4), and is actually suggested by String Field Theory [20]. It can also be extended in a relatively simple fashion to (A)dS backgrounds, and similar results apply to its fermionic analog of [18, 19].

The role of the unconstrained gauge symmetry in the interactions of higher-spin gauge fields is less clear at the moment, but there are clues that off-shell they will eventually make use of it. To wit, while the four-dimensional Vasiliev construction of [11] was based on a generalization of the two-component formalism for gravity, and as a result is strictly tied to the Fronsdal form, this is not necessarily the case for the more recent construction of [13]. As stressed in [5], in

some respects the new formulation of [13] can be regarded as a step forward in the direction of an off-shell formulation, and indeed it allows to drop the trace conditions. When this is done [5], at the free level one recovers very nicely the local compensator equations of [18, 19]. At the interacting level, however, this choice entails some subtleties that are spelled out in detail in [5, 4], and further work is required to settle the issue of its consistency, although a forthcoming microscopic analysis of the Vasiliev equations lends further, independent support to the role of the unconstrained symmetry [23]. Recently, additional evidence for the potential role of the unconstrained symmetry for the off-shell description of higher-spin gauge fields was also provided in [24].

Complete off-shell formulations for free symmetric tensors were already introduced some time ago by Pashnev and Tsulaia [21]. Like their more recent fermionic counterparts of [22], these constructions rest on the BRST formalism and describe spin - s symmetric tensors via additional fields whose number grows proportionally to s . The resulting free systems are rather complicated, but nonetheless in [19] it was shown that a judicious elimination of most of the additional fields reproduces the geometric equations (1.3) and (1.4) in the compensator form of [18, 19]. The present letter is devoted to displaying far simpler constructions, minimal Lagrangians where the trace constraints of the conventional Fronsdal formulation are eliminated adding *only two* fields, the compensator α and a single spin - $(s - 4)$ symmetric tensor β playing the role of a Lagrange multiplier. The next Section reviews briefly the results of [15, 18, 19], while the two remaining Sections are devoted, respectively, to the minimal bosonic Lagrangians and to their fermionic counterparts.

2 GEOMETRIC AND LOCAL COMPENSATOR EQUATIONS

As anticipated in the Introduction, in this paper we restrict our attention to an important class of higher-spin fields, totally symmetric (spinor-)tensors, a choice that has the virtue of allowing a relatively handy discussion. The Fronsdal equations are

$$\mathcal{F} \equiv \square \varphi - \partial \partial \cdot \varphi + \partial^2 \varphi' = 0 , \quad (2.1)$$

where \mathcal{F} will be often referred to as the Fronsdal operator. For spin one and two, eqs. (2.1) reduce to the Maxwell equation for the vector potential A_μ and to the linearized Einstein equation for the metric fluctuation $h_{\mu\nu}$, while novelties begin to emerge for spin 3.

Let us pause briefly to explain our notation. In this letter, as in [15, 18, 19], primes (or bracketed suffixes) denote traces, while all indices carried by the symmetric tensors $\varphi_{\mu_1 \dots \mu_s}$ and $\Lambda_{\mu_1 \dots \mu_{s-1}}$, by the metric tensor $\eta_{\mu\nu}$ or by derivatives are left implicit. In order to fully profit from this shorthand notation, where all terms are meant to be totally symmetrized so that, for instance, $\partial \varphi$ stands for $\partial_{\mu_1} \varphi_{\mu_2 \dots \mu_{s+1}} + \dots$, one need only get accustomed to a few rules, that is convenient to display again, correcting also a misprint in [18]:

$$\begin{aligned} (\partial^p \varphi)' &= \square \partial^{p-2} \varphi + 2 \partial^{p-1} \partial \cdot \varphi + \partial^p \varphi' , \\ \partial^p \partial^q &= \binom{p+q}{p} \partial^{p+q} , \\ \partial \cdot (\partial^p \varphi) &= \square \partial^{p-1} \varphi + \partial^p \partial \cdot \varphi , \\ \partial \cdot \eta^k &= \partial \eta^{k-1} , \\ (\eta^k \varphi)' &= [D + 2(s + k - 1)] \eta^{k-1} \varphi + \eta^k \varphi' . \end{aligned} \quad (2.2)$$

We shall work throughout in D dimensions, with a mostly positive space-time signature, and in this notation the Fronsdal Lagrangian is simply

$$\mathcal{L}_0 = \frac{1}{2} \varphi \left(\mathcal{F} - \frac{1}{2} \eta \mathcal{F}' \right) , \quad (2.3)$$

where, as will be always the case in the following, evident contractions between different fields in Lorentz invariant monomials are left implicit. For instance, here φ is implicitly contracted with the Einstein-like tensor $\mathcal{F} - \frac{1}{2} \eta \mathcal{F}'$.

This formulation rests crucially on two constraints, that first emerge for spin 3 and 4. The first concerns the gauge parameter, Λ , that enters the gauge transformation of φ in the conventional fashion, as $\delta \varphi = \partial \Lambda$, but is to be *traceless* in order to guarantee the gauge invariance of (2.1), since

$$\delta \mathcal{F} = 3 \partial^3 \Lambda' . \quad (2.4)$$

The second concerns the gauge field φ itself, that is to be *doubly traceless*. This peculiar restriction originates from the “anomalous” Bianchi identity for the Fronsdal operator,

$$\partial \cdot \mathcal{F} - \frac{1}{2} \partial \mathcal{F}' = - \frac{3}{2} \partial^3 \varphi'' , \quad (2.5)$$

since indeed, even with a traceless Λ , and up to partial integrations that will be always left implicit in the following, the Lagrangian (2.3) *is not* gauge invariant, but varies into

$$\delta \mathcal{L}_0 = - \frac{s}{2} \Lambda \left(\partial \cdot \mathcal{F} - \frac{1}{2} \partial \mathcal{F}' \right) . \quad (2.6)$$

In [15] we showed that, making use of the gauge field φ only, one can build a sequence of pseudo-differential analogs of \mathcal{F} ,

$$\mathcal{F}^{(k+1)} = \mathcal{F}^{(k)} + \frac{1}{(k+1)(2k+1)} \frac{\partial^2}{\square} \mathcal{F}^{(k)'} - \frac{1}{k+1} \frac{\partial}{\square} \partial \cdot \mathcal{F}^{(k)} , \quad (2.7)$$

such that

$$\delta \mathcal{F}^{(k)} = (2k+1) \frac{\partial^{2k+1}}{\square^{k-1}} \Lambda^{[k]} . \quad (2.8)$$

For spin $s = 2k-1$ or $s = 2k$, $\mathcal{F}^{(k)} = 0$ is thus the minimal fully gauge invariant modification of the Fronsdal equation. Interestingly, this result is directly linked to the higher-spin curvatures \mathcal{R} introduced by de Wit and Freedman [16], and indeed the fully gauge invariant equations can be written in the more suggestive *geometric* fashion [18]

$$\frac{1}{\square^p} \partial \cdot \mathcal{R}^{[p]}_{;\alpha_1 \dots \alpha_{2p+1}} = 0 , \quad (2.9)$$

for odd spins $s = 2p+1$, and

$$\frac{1}{\square^{p-1}} \mathcal{R}^{[p]}_{;\alpha_1 \dots \alpha_{2p}} = 0 , \quad (2.10)$$

for even spins $s = 2p$.

In this formulation gauge invariance is manifest, and does not require any constraints on the gauge parameter, but the equivalence to the Fronsdal formulation entails a few subtleties, that are spelled out in [18]. In addition, the Bianchi identity is also modified, so that

$$\partial \cdot \mathcal{F}^{(k)} - \frac{1}{2k} \partial \mathcal{F}^{(k)'} = - \left(1 + \frac{1}{2k} \right) \frac{\partial^{2k+1}}{\square^{k-1}} \varphi^{[k+1]} , \quad (2.11)$$

and this suffices to show that, for every given spin, there exists a lowest value of k such that the generalized Einstein tensors

$$\mathcal{G}^{(k)} = \sum_{p \leq k} \frac{(-1)^p}{2^p p! \binom{k}{p}} \eta^p \mathcal{F}^{(k)[p]} \quad (2.12)$$

are divergence-free, and hence the Lagrangians

$$\mathcal{L} = \frac{1}{2} \varphi \mathcal{G}^{(k)} \quad (2.13)$$

are fully gauge invariant.

In [15] we also showed that the geometric equations (2.9) and (2.10) can be always turned into the form

$$\mathcal{F} = 3 \partial^3 \mathcal{H} , \quad (2.14)$$

with \mathcal{H} a *non-local construct* of \mathcal{F} , bound to transform as $\delta \mathcal{H} = \Lambda'$ under a gauge transformation. In other words, \mathcal{H} behaves as a *compensator* for the trace of the gauge parameter.

In [18, 19], drawing also from String Field Theory [20], we explored the implications of allowing in the theory an *independent field* α , denoted as the “compensator” and such that $\delta \alpha = \Lambda'$. The key result of this analysis was that with φ and α one can arrive at the two local field equations

$$\mathcal{F} = 3 \partial^3 \alpha , \quad (2.15)$$

$$\varphi'' = 4 \partial \cdot \alpha + \partial \alpha' , \quad (2.16)$$

that can also be obtained truncating the bosonic triplet of [26, 21], are nicely consistent with the Bianchi identity (2.5), but *are not* Lagrangian. As shown in [18] (see also [25]), from these one can readily recover the non-local geometric equations (2.9) and (2.10) building a sequence of equations for the non-local extensions $\mathcal{F}^{(k)}$ of (2.7), since eq. (2.15) implies that

$$\mathcal{F}^{(k)} = (2k + 1) \frac{\partial^{2k+1}}{\square^k} \alpha^{[k]} , \quad (2.17)$$

and finally, after the minimal number of iterations needed to produce a trace of α not allowed for a given spin s , eqs. (2.9) and (2.10).

In a similar fashion starting from the fermionic Fang-Fronsdal operator

$$\mathcal{S} = i (\not{\partial} \psi - \partial \not{\psi}) , \quad (2.18)$$

one can define a sequence of non-local extensions of \mathcal{S} , directly linked to the bosonic ones according to

$$\mathcal{S}_{n+1/2}^{(k)} - \frac{1}{2k} \frac{\partial}{\square} \not{\partial} \mathcal{S}_{n+1/2}^{(k)} = i \frac{\not{\partial}}{\square} \mathcal{F}_n^{(k)}(\psi) , \quad (2.19)$$

with $\mathcal{F}_n^{(k)}(\psi)$ a non-local extension of the Fronsdal operator for the spinor-tensor ψ , and arrive eventually at non-local geometric equations. Even in this case, non-Lagrangian equations for spin - $(n + 1/2)$ spinor-tensors $\psi_{\mu_1 \dots \mu_n}$ involving a single spin - $(n - 3/2)$ compensator $\xi_{\mu_1 \dots \mu_{n-2}}$ were obtained in [18, 19]. In flat space they read

$$\begin{aligned} \mathcal{S} &= -2i \partial^2 \xi , \\ \not{\psi}' &= 2 \partial \cdot \xi + \partial \xi' + \not{\partial} \not{\xi} , \end{aligned} \quad (2.20)$$

and can be obtained truncating the fermionic triplet introduced in [18].

3 LOCAL LAGRANGIANS FOR UNCONSTRAINED BOSONS

In the previous Section we have reviewed the salient features of the non-local geometric equations of [15] and their relation with the non-Lagrangian equations of [18, 19]. As shown in [19], the latter also follow, albeit in a somewhat indirect fashion, from the BRST construction of Pashnev and Tsulaia [21].

In this Section we would like to present a simple alternative: local Lagrangians for unconstrained spin - s tensor fields φ that involve at most *two* additional fields, the spin - $(s - 3)$ compensator α of [18, 19], and an additional spin - $(s - 4)$ field β that acts as a Lagrange multiplier for the relation between the double trace φ'' and the compensator α in eq. (2.16). These “minimal” Lagrangians are closely related to the geometric equations (2.9) and (2.10) of [15] via the compensator system (2.15) and (2.16), and for spin $s = 3$ reduce to the result already presented in [15].

A minimal gauge invariant Lagrangian for spin - s symmetric tensors φ can be nicely determined resorting to the familiar Noether procedure, that allows one to deal with this problem in a systematic fashion and has the additional virtue of clarifying the origin of the difficulty met in a naive approach beyond the spin - 3 case. Let us therefore begin by considering the Fronsdal expression (2.3), now written for a field φ *not subject* to any trace constraints, and let us vary it without enforcing any constraints on the gauge parameter Λ . The resulting complete variation,

$$\begin{aligned} \delta \mathcal{L}_0 = & -3 \binom{s}{4} \varphi'' \partial \cdot \partial \cdot \partial \cdot \Lambda - 9 \binom{s}{4} \partial \cdot \partial \cdot \varphi' \partial \cdot \Lambda' + \frac{15}{2} \binom{s}{5} \partial \cdot \partial \cdot \partial \cdot \varphi' \Lambda'' \\ & + \Lambda' \binom{s}{3} \left\{ \frac{3}{4} \partial \cdot \mathcal{F}' - \frac{3}{2} \partial \cdot \partial \cdot \partial \cdot \varphi + \frac{9}{4} \square \partial \cdot \varphi' \right\}, \end{aligned} \quad (3.1)$$

comprises a number of terms depending on Λ' , that can be canceled adding

$$\begin{aligned} \mathcal{L}_1 = & -\alpha \binom{s}{3} \left\{ \frac{3}{4} \partial \cdot \mathcal{F}' - \frac{3}{2} \partial \cdot \partial \cdot \partial \cdot \varphi + \frac{9}{4} \square \partial \cdot \varphi' \right\} \\ & + 9 \binom{s}{4} \partial \cdot \alpha \partial \cdot \partial \cdot \varphi' - \frac{15}{2} \binom{s}{5} \alpha' \partial \cdot \partial \cdot \partial \cdot \varphi', \end{aligned} \quad (3.2)$$

that depends linearly on the compensator α .

Additional terms depending on Λ' now present themselves in the resulting variation of $\mathcal{L}_0 + \mathcal{L}_1$, but can be eliminated adding

$$\begin{aligned} \mathcal{L}_2 = & \frac{9}{4} \binom{s}{3} \alpha \square^2 \alpha - 27 \binom{s}{4} \partial \cdot \alpha \square \partial \cdot \alpha + 45 \binom{s}{5} (\partial \cdot \partial \cdot \alpha)^2 \\ & + \frac{45}{2} \binom{s}{5} \partial \cdot \partial \cdot \alpha \square \alpha' - 45 \binom{s}{6} \partial \cdot \partial \cdot \partial \cdot \alpha \partial \cdot \alpha', \end{aligned} \quad (3.3)$$

that is quadratic in α , so that the final remainder is

$$\delta \{ \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 \} = -3 \binom{s}{4} \{ \varphi'' - 4 \partial \cdot \alpha - \partial \alpha' \} \partial \cdot \partial \cdot \partial \cdot \Lambda. \quad (3.4)$$

These terms vanish for $s < 4$, and are proportional to the gauge invariant expression given by eq. (2.16). A fully gauge invariant unconstrained Lagrangian is thus finally obtained introducing, from spin $s = 4$ onwards, the single additional term

$$\mathcal{L}_3 = 3 \binom{s}{4} \beta (\varphi'' - 4 \partial \cdot \alpha - \partial \alpha'), \quad (3.5)$$

where the spin - $(s - 4)$ *Lagrange multiplier* β transforms as $\delta\beta = \partial \cdot \partial \cdot \partial \cdot \Lambda$.

Summarizing, the complete Lagrangians for unconstrained spin - s totally symmetric tensors are

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \varphi \left(\mathcal{F} - \frac{1}{2} \eta \mathcal{F}' \right) - \binom{s}{3} \alpha \left\{ \frac{3}{4} \partial \cdot \mathcal{F}' - \frac{3}{2} \partial \cdot \partial \cdot \partial \cdot \varphi + \frac{9}{4} \square \partial \cdot \varphi' \right\} \\ & + 9 \binom{s}{4} \partial \cdot \alpha \partial \cdot \partial \cdot \varphi' - \frac{15}{2} \binom{s}{5} \alpha' \partial \cdot \partial \cdot \partial \cdot \varphi' + \frac{9}{4} \binom{s}{3} \alpha \square^2 \alpha \\ & - 27 \binom{s}{4} \partial \cdot \alpha \square \partial \cdot \alpha + 45 \binom{s}{5} (\partial \cdot \partial \cdot \alpha)^2 + \frac{45}{2} \binom{s}{5} \partial \cdot \partial \cdot \alpha \square \alpha' \\ & - 45 \binom{s}{6} \partial \cdot \partial \cdot \partial \cdot \alpha \partial \cdot \alpha' + 3 \binom{s}{4} \beta (\varphi'' - 4 \partial \cdot \alpha - \partial \alpha') , \end{aligned} \quad (3.6)$$

and are invariant under the gauge transformations

$$\begin{aligned} \delta \varphi &= \partial \Lambda , \\ \delta \alpha &= \Lambda' , \\ \delta \beta &= \partial \cdot \partial \cdot \partial \cdot \Lambda . \end{aligned} \quad (3.7)$$

We can now move on to clarify the connection with the non-Lagrangian system of eqs. (2.15) and (2.16). The starting point are the field equations determined by (3.6),

$$\begin{aligned} \varphi : \quad \mathcal{F} - 3 \partial^3 \alpha - \frac{1}{2} \eta (\mathcal{F}' - \frac{1}{2} \partial^2 \varphi'' - 3 \square \partial \alpha - 4 \partial^2 \partial \cdot \alpha - \frac{3}{2} \partial^3 \alpha') \\ + \eta^2 (\beta + \frac{1}{2} \partial \partial \cdot \partial \cdot \alpha + \square \partial \cdot \alpha - \frac{1}{2} \partial \cdot \partial \cdot \varphi') = 0 , \end{aligned} \quad (3.8)$$

$$\beta : \quad \varphi'' - 4 \partial \cdot \alpha - \partial \alpha' = 0 , \quad (3.9)$$

$$\begin{aligned} \alpha : \quad 6 \square^2 \alpha + 18 \square \partial \partial \cdot \alpha + 12 \partial^2 \partial \cdot \partial \cdot \alpha + 3 \square \partial^2 \alpha' + 3 \partial^3 \partial \cdot \alpha' \\ - 3 \partial \partial \cdot \partial \cdot \varphi' - \partial \cdot \mathcal{F}' + 2 \partial \cdot \partial \cdot \partial \cdot \varphi - 3 \square \partial \cdot \varphi' + 4 \partial \beta \\ + \eta (3 \square \partial \cdot \partial \cdot \alpha + \partial \partial \cdot \partial \cdot \partial \cdot \alpha - \partial \cdot \partial \cdot \partial \cdot \varphi' + 2 \partial \cdot \beta) = 0 , \end{aligned} \quad (3.10)$$

and the issue is to show their equivalence to eq. (2.15). A general argument to this effect can be built as follows.

Let us begin by noticing that, when β is on-shell, *i.e.* when eq. (3.9) is enforced, the φ equation becomes of the form

$$\mathcal{A} - \frac{1}{2} \eta \mathcal{A}' + \eta^2 \mathcal{C} = 0 , \quad (3.11)$$

where

$$\begin{aligned} \mathcal{A} &= \mathcal{F} - 3 \partial^3 \alpha , \\ \mathcal{C} &= \beta + \frac{1}{2} \partial \partial \cdot \partial \cdot \alpha + \square \partial \cdot \alpha - \frac{1}{2} \partial \cdot \partial \cdot \varphi' , \end{aligned} \quad (3.12)$$

and that, under the same condition, the double trace of \mathcal{A} vanishes *identically*. One can then take successive traces of (3.11): whereas the first relates \mathcal{A}' to \mathcal{C} and \mathcal{C}' , the higher ones yield

relations of the form

$$(\eta^2 \mathcal{C})^{[k]} = \eta^2 \mathcal{C}^{[k]} + \sum_{i=1}^k \rho_{2i+1} \eta \mathcal{C}^{[k-1]} + \sum_{i \leq j=2}^k \rho_{2i-1} \rho_{2j} \mathcal{C}^{[k-2]} = 0. \quad (3.13)$$

It should be appreciated that the $\mathcal{C}^{[i]}$ are all independent and do not vanish identically. As a result, these never reduce to trivial identities, since the coefficients $\rho_k \equiv D + 2(s - k)$ are positive. Therefore, denoting with p the integer part of $\frac{s-4}{2}$ and taking $p + 2$ traces of (3.11) gives

$$\sum_{i \leq j=2}^{p+2} \rho_{2i-1} \rho_{2j} \mathcal{C}^{[p]} = 0, \quad (3.14)$$

and hence, finally, $\mathcal{C}^{[p]} = 0$. Making use of this relation in the $[p + 1]$ -th trace, and working backwards, one can convince oneself that *on shell all traces of \mathcal{C} , including \mathcal{C} itself, vanish*. In the first trace of (3.11), this result gives $\mathcal{A}' = 0$, and then finally eq. (3.11) turns into the desired form (2.15)³.

As we have seen, the field equation for the Lagrange multiplier β is the condition that the double trace of the dynamical field be pure gauge, and plays a crucial role in linking these Lagrangian equations to the geometrical ones. On the other hand, (3.10) has not played any role so far, in particular in the relation between the local Lagrangian (3.6) and the higher-spin geometry. There is a reason for this: (3.10) is a *consequence* of the field equations for φ and β . Indeed, taking the divergence of eq. (3.8) and using eq. (3.9) in the result, one arrives at an expression proportional to (3.10). More precisely, indicating with $\mathcal{G}_{\varphi,\beta}(\alpha)$ the field operator for the compensator α , one can see that

$$\partial \cdot \{ \mathcal{A} - \frac{1}{2} \eta \mathcal{A}' + \eta^2 \mathcal{C} \} = \frac{\eta}{4} \mathcal{G}_{\varphi,\beta}(\alpha). \quad (3.15)$$

It is then clear that if φ and β satisfy their field equations, $\mathcal{G}_{\varphi,\beta}(\alpha)$ is forced to vanish. That is to say, α is forced to satisfy its field equation as well.

Actually, the role of the field equation for the compensator α can be better appreciated if the dynamical field φ is coupled to an external source \mathcal{J} . In the Fronsdal case, the Lagrangian equation is

$$\mathcal{F} - \frac{1}{2} \eta \mathcal{F}' = \mathcal{J}, \quad (3.16)$$

and its divergence gives

$$-\frac{1}{2} \eta \partial \cdot \mathcal{F}' = \partial \cdot \mathcal{J}. \quad (3.17)$$

Hence, while in the Maxwell and Einstein cases $\partial \cdot \mathcal{F}'$ vanishes identically, so that the sources must be *divergence free*, in the conventional formulation for spin 3 or higher only the *traceless part of the divergence* is forced to vanish. In [6] it was shown that even this weaker condition suffices to ensure that only physical polarizations contribute to the exchange of quanta between sources. On the other hand, if an external source is introduced in our Lagrangian (3.6) via the standard coupling $\varphi \cdot \mathcal{J}$, the field equations become

$$\mathcal{A} - \frac{1}{2} \eta \mathcal{B} + \eta^2 \mathcal{C} = \mathcal{J}, \quad (3.18)$$

³ $s = 2$ and $D = 2$ is a well-known exception, since in that case the trace of $\mathcal{A} - \frac{1}{2} \eta \mathcal{A}'$ vanishes identically, giving no indications on \mathcal{A}' .

$$\mathcal{G}_{\varphi,\beta}(\alpha) = 0, \quad (3.19)$$

$$\varphi'' - 4\partial \cdot \alpha - \partial \alpha' = 0, \quad (3.20)$$

where \mathcal{B} is defined by comparing with eq. (3.8). Combining the divergence of (3.18) with (3.20) then yields

$$\frac{\eta}{4} \mathcal{G}_{\varphi,\beta}(\alpha) = \partial \cdot \mathcal{J}, \quad (3.21)$$

a result apparently similar to Fronsda's [6]. However, now the full Lagrangian system implies that, when the compensator α satisfies its field equation, the coupling can only be consistent if the current \mathcal{J} is *divergence free*. Incidentally, this is just the expected Noether constraint for a source related to the gauge symmetry of the theory described by (3.6).

4 LOCAL LAGRANGIANS FOR UNCONSTRAINED FERMIONS

One can repeat the previous steps almost verbatim for fermion fields. Here the starting point is provided by the Fang-Fronsdal equation for a symmetric spin - $(n + 1/2)$ spinor-tensor ψ ,

$$\mathcal{S} \equiv i (\not{\partial} \psi - \partial \psi) = 0, \quad (4.1)$$

that is invariant under the gauge transformation $\delta \psi = \partial \epsilon$ only if the gauge parameter is γ - *traceless*, since

$$\delta \mathcal{S} = -2i \partial^2 \not{\epsilon}. \quad (4.2)$$

In a similar fashion, the corresponding Lagrangian

$$\mathcal{L} = \frac{1}{2} \bar{\psi} \left(\mathcal{S} - \frac{1}{2} \gamma \not{\mathcal{S}} - \frac{1}{2} \eta \mathcal{S}' \right) + h.c. \quad (4.3)$$

is gauge invariant only if the gauge field ψ is *triply* γ - *traceless*, on account of the “anomalous” Bianchi identity

$$\partial \cdot \mathcal{S} - \frac{1}{2} \partial \mathcal{S}' - \frac{1}{2} \not{\partial} \not{\mathcal{S}} = i \partial^2 \psi'. \quad (4.4)$$

In this case one begins by considering the Lagrangian (4.3), written however for an *unconstrained* Fermi field ψ of spin $n + 1/2$, and varied with an *unconstrained* gauge parameter ϵ . The resulting variation is then

$$\begin{aligned} \delta \mathcal{L}_0 = & - \frac{3i}{2} \binom{n}{3} \partial \cdot \partial \cdot \bar{\epsilon} \psi' + \frac{3i}{4} \binom{n}{3} \bar{\epsilon}' \not{\partial} \partial \cdot \psi' - 3i \binom{n}{3} \bar{\epsilon}' \partial \cdot \partial \cdot \psi \\ & - \frac{3i}{4} \binom{n}{3} \bar{\epsilon}' \square \psi' + 3i \binom{n}{4} \partial \cdot \bar{\epsilon}' \partial \cdot \psi' + 2i \binom{n}{2} \bar{\not{\epsilon}} \partial \cdot \partial \cdot \psi \\ & - 2i \binom{n}{2} \bar{\not{\epsilon}} \not{\partial} \partial \cdot \psi - i \binom{n}{2} \bar{\not{\epsilon}} \square \psi' + \frac{9i}{2} \binom{n}{3} \partial \cdot \bar{\not{\epsilon}} \partial \cdot \psi' \\ & - 3i \binom{n}{4} \bar{\not{\epsilon}}' \partial \cdot \partial \cdot \psi' + h.c.. \end{aligned} \quad (4.5)$$

In complete analogy with the bosonic case, all terms involving the γ - trace $\not{\epsilon}$ of the gauge parameter can be canceled by additional terms linear in the compensator field ξ , that are collected

in

$$\begin{aligned}
\mathcal{L}_1 = & -\frac{3i}{4} \binom{n}{3} \bar{\xi} \not{\partial} \partial \cdot \psi' + 3i \binom{n}{3} \bar{\xi} \partial \cdot \partial \cdot \psi + \frac{3i}{4} \binom{n}{3} \bar{\xi} \square \psi' \\
& - 3i \binom{n}{4} \partial \cdot \bar{\xi} \partial \cdot \psi' - 2i \binom{n}{2} \bar{\xi} \partial \cdot \partial \cdot \psi + 2i \binom{n}{2} \bar{\xi} \not{\partial} \partial \cdot \psi \\
& + i \binom{n}{2} \bar{\xi} \square \psi' - \frac{9i}{2} \binom{n}{3} \partial \cdot \bar{\xi} \partial \cdot \psi' + 3i \binom{n}{4} \bar{\xi}' \partial \cdot \partial \cdot \psi' + h.c..
\end{aligned} \tag{4.6}$$

Additional terms depending on $\not{\epsilon}$ generated by the variation of \mathcal{L}_1 can then be eliminated adding

$$\begin{aligned}
\mathcal{L}_2 = & -\frac{15i}{2} \binom{n}{3} \bar{\xi} \square \partial \cdot \xi - i \binom{n}{2} \bar{\xi} \square \not{\partial} \xi + 3i \binom{n}{3} \partial \cdot \bar{\xi} \not{\partial} \partial \cdot \xi \\
& + 18i \binom{n}{4} \partial \cdot \bar{\xi} \partial \cdot \partial \cdot \xi + 6i \binom{n}{4} \partial \cdot \bar{\xi} \square \xi' \\
& - 15i \binom{n}{5} \partial \cdot \partial \cdot \bar{\xi} \partial \cdot \xi' + h.c.,
\end{aligned} \tag{4.7}$$

and the total variation is finally

$$\delta \{ \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 \} = -\frac{3i}{2} \binom{n}{3} \partial \cdot \partial \cdot \bar{\epsilon} (\psi' - 2 \partial \cdot \xi - \not{\partial} \xi - \partial \xi') + h.c.. \tag{4.8}$$

This residual contribution is proportional to the constraint relating ψ' to the compensator in eqs. (2.20). One can finally introduce a Lagrange multiplier field λ , a spinor-tensor of spin $n - 5/2$ such that $\delta \lambda = \partial \cdot \partial \cdot \epsilon$, to compensate (4.8) by the additional term

$$\mathcal{L}_3 = \frac{3i}{2} \binom{n}{3} \bar{\lambda} (\psi' - 2 \partial \cdot \xi - \not{\partial} \xi - \partial \xi') + h.c.. \tag{4.9}$$

Summarizing, the complete Lagrangian for an *unconstrained* spin - $(n + 1/2)$ spinor-tensor is

$$\begin{aligned}
\mathcal{L} = & \frac{1}{2} \bar{\psi} \left(\mathcal{S} - \frac{1}{2} \gamma \not{\mathcal{S}} - \frac{1}{2} \eta \mathcal{S}' \right) \\
& - \frac{3i}{4} \binom{n}{3} \bar{\xi} \not{\partial} \partial \cdot \psi' + 3i \binom{n}{3} \bar{\xi} \partial \cdot \partial \cdot \psi + \frac{3i}{4} \binom{n}{3} \bar{\xi} \square \psi' \\
& - 3i \binom{n}{4} \partial \cdot \bar{\xi} \partial \cdot \psi' - 2i \binom{n}{2} \bar{\xi} \partial \cdot \partial \cdot \psi + 2i \binom{n}{2} \bar{\xi} \not{\partial} \partial \cdot \psi \\
& + i \binom{n}{2} \bar{\xi} \square \psi' - \frac{9i}{2} \binom{n}{3} \partial \cdot \bar{\xi} \partial \cdot \psi' + 3i \binom{n}{4} \bar{\xi}' \partial \cdot \partial \cdot \psi' \\
& - \frac{15i}{2} \binom{n}{3} \bar{\xi} \square \partial \cdot \xi - i \binom{n}{2} \bar{\xi} \square \not{\partial} \xi + 3i \binom{n}{3} \partial \cdot \bar{\xi} \not{\partial} \partial \cdot \xi \\
& + 18i \binom{n}{4} \partial \cdot \bar{\xi} \partial \cdot \partial \cdot \xi + 6i \binom{n}{4} \partial \cdot \bar{\xi} \square \xi' - 15i \binom{n}{5} \partial \cdot \partial \cdot \bar{\xi} \partial \cdot \xi' \\
& + \frac{3i}{2} \binom{n}{3} \bar{\lambda} (\psi' - 2 \partial \cdot \xi - \not{\partial} \xi - \partial \xi') + h.c.,
\end{aligned} \tag{4.10}$$

and is invariant under the gauge transformations

$$\begin{aligned}
\delta \psi &= \partial \epsilon, \\
\delta \xi &= \not{\epsilon}, \\
\delta \lambda &= \partial \cdot \partial \cdot \epsilon.
\end{aligned} \tag{4.11}$$

The corresponding field equations are:

$$\begin{aligned} \bar{\psi} : \quad & -i\mathcal{S} + 2\partial^2\xi - \frac{1}{2}\eta(-i\mathcal{S}' + \frac{1}{2}\partial\psi' - \frac{1}{2}\partial\partial\xi + 2\Box\xi + 3\partial\partial\cdot\xi + \partial^2\xi') \\ & - \frac{1}{2}\gamma(-i\mathcal{S} + 2\partial^2\xi + 2\partial\partial\xi) + \frac{1}{4}\gamma\eta(\partial\cdot\psi' - \Box\xi - \partial\partial\cdot\xi - 2\lambda) = 0, \end{aligned} \quad (4.12)$$

$$\bar{\lambda} : \quad \psi' - 2\partial\cdot\xi - \partial\xi' = 0, \quad (4.13)$$

$$\begin{aligned} \bar{\xi} : \quad & \Box\psi' + 2\partial\partial\cdot\psi - 2\partial\cdot\partial\cdot\psi + \frac{3}{2}\partial\partial\cdot\psi' - 2\Box\partial\xi - \frac{5}{2}\Box\partial\xi - 2\partial\partial\partial\cdot\xi \\ & - 3\partial^2\partial\cdot\xi - \partial\lambda + \eta(\frac{1}{2}\partial\cdot\partial\cdot\psi' - \Box\partial\cdot\xi - \frac{1}{2}\partial\partial\cdot\partial\cdot\xi - \partial\cdot\lambda) \\ & + \gamma(-\frac{1}{4}\partial\partial\cdot\psi' + \partial\cdot\partial\cdot\psi + \frac{1}{4}\Box\psi' + \frac{1}{4}\partial\partial\cdot\psi' \\ & - \frac{5}{2}\Box\partial\cdot\xi - \frac{3}{2}\partial\partial\cdot\partial\cdot\xi - \frac{1}{2}\Box\partial\xi' - \frac{1}{2}\partial^2\partial\cdot\xi' - \frac{1}{2}\partial\lambda) = 0. \end{aligned} \quad (4.14)$$

As in the bosonic case, we can now relate them to the simple non-Lagrangian system (2.20). The basic observation is to recognize that, when (4.13) is satisfied, the equation for ψ takes the form

$$\mathcal{W} - \frac{1}{2}\gamma\mathcal{W} - \frac{1}{2}\eta\mathcal{W}' + \frac{i}{4}\eta\gamma\mathcal{Z} = 0, \quad (4.15)$$

where

$$\begin{aligned} \mathcal{W} &= \mathcal{S} + 2i\partial^2\xi, \\ \mathcal{Z} &= \partial\cdot\psi' - \Box\xi - \partial\partial\cdot\xi - 2\lambda. \end{aligned} \quad (4.16)$$

Moreover, under the same conditions the triple γ - trace of \mathcal{W} vanishes. It is then possible to rephrase the iterative argument presented for bosonic fields: if p is the integer part of $\frac{s-3}{2}$, where $s \geq 3$, taking $(p+1)$ successive traces of (4.15) one arrives at the condition that the highest γ - trace vanish:

$$\gamma \cdot \left(\frac{1}{4}\eta\gamma\mathcal{Z} \right)^{[p+1]} = 0. \quad (4.17)$$

Inserting (4.17) in the lower γ - traces of (4.15), one can recursively show that *all* γ - traces of \mathcal{Z} vanish as well. Consequently, the first trace and the γ - trace of (4.15) imply that $\mathcal{W}' = 0$ and $\mathcal{W} = 0$, and in conclusion the Lagrangian (4.10) leads indeed to the local compensator equations (2.20). In complete analogy with the bosonic case, making use of (4.13) one can show that the field equation (4.14) for the compensator ξ is proportional to the divergence of (4.12).

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